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AN EPQ MODEL FOR DETERIORATING ITEMS WITH DEMAND AND TIME VARYING DETERIORATION WITH SHORTAGES

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ABSTRACT

The paper presents a production model, which was developed for its deteriorating items with demand rate and time dependent deterioration rate, production rate in demand dependent and greater than the demand rate. The shortages are allowed and completely backlogged. Also, the optimal solution of the model is got, for a numerical example after some adjustments in the derived solution. The carrying out of the sensitivity analysis is also done, in order to examine the effects of changes in model parameters on the optimal solution.

KEYWORDS: With Shortages, Deteriorating Items, Sensitivity Analysis, Demand Rate

INTRODUCTION

In this paper, an inventory models create lot of interest, due to their ready applicability for many practical situations, arising in the place like food and vegetable markets, pharmaceutical and chemical industries, cement industries, market yards, oil exploration industries, etc. The two most important decision variables in any Economic Production Quantity (EPQ) model are the production run time and the optimal quantity to be produced. Production inventory models are meant to find the optimal values of these decision variables, while minimizing the total cost of production. The inventory models efficiency depends upon the suitable assumptions, made on the constituent components of the model. The chief components of the model are: (1) replenishment (2) demand pattern and (3) life time of commodity. Several production level inventory models have been developed and analyzed, with various assumptions on demand rate and life time of the commodity. It is customary to consider that, the replenishment in many production inventory models has finite or infinite rate.

Ritchie (1984), Dave (1986), Urban (1992), Chakrabarti and Chaudhuri (1997), Venkata Subbaiah, *et al.* (2004), Skouri, *et al. (*2009*)* and Misra, *et al.* (2011) have developed inventory models, having infinite production rate. Deb and Chaudhuri (1986), Madam and Phaujdar (1989), Sana, *et al.*(2004), Srinivasa Rao and Begum (2007), Uma Maheswara Rao, *et al. (*2010*)* and Darker and Moon (2011), studied the finite rate of production. Nobel and Headed (2000), considered a stochastic inventory model, with two discrete production systems. Two different rates of replenishment, in one inventory system were also studied (Perumal and Arivarignan (2002); Sen and Chakrabarty (2007). Sridevi, *et al. (*2010*),* developed an inventory model for deteriorating items, with random replenishment. Eassey, *et al.* (2012), developed an inventory model for deteriorating items with stock dependent production rate and Weibull decay. Recently, Lakshman Rao *et al.* (2015), developed studies on some inventory model, for deteriorating items with Weibull replenishment and generalized Pareto decay, having demand as a function on hand inventory.

In all these papers they assumed that, the production rate is independent. Very little work has been reported in literature, regarding inventory models for deteriorating items with dependent demand rate, which are useful for developing optimal ordering policies, using the resources more efficiently and effectively. Hence, in this paper, we develop an inventory model for deteriorating items with dependent demand rate, having Weibull rate of decay and time dependent demand. Here, it is assumed that, the production rate is linearly decreasing function of the demand rate. It also includes finite rate of production, as a particular case. The Weibull rate of deterioration includes constant, increasing and decreasing rates of decay. The time dependent demand rate also includes several forms of demand, since it is considered that, it is of the form $\lambda(t)=\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$, where 'n' is the indexing parameter.

The instantaneous sate of inventory is the result of differential equations. The total cost function is the product of, suitable cost consideration. The minimization of the total cost will result in, the optimal ordering quantity, optimal production downtime and optimal production uptime. The sensitivity of the model, with respect to cost and parameters is also studied.

ECONOMIC MODEL WITH SHORTAGES

In this paper, we develop the economic production quantity model, with shortages. Here, it is assumed that, the production is governed by various factors influenced by random causes. The life of commodity is random, and follows a probability distribution.

Notations

Then following notations are used for developing the model.

A: ordering cost

C: per of unit production cost the items

h: inventory holding cost per unit time

π: shortage cost per unit time

Q: Total quantity of items produced in one cycle

T: length of the cycle

$\lambda(t)$:demand rate at any time 't'

K: Total profit per unit time in the system

K(t): production rate at any time 't'

(α,β,γ) :Deterioration rate parameters

n: pattern index

r: Total demand during the cycle

(θ,η): Demand rate parameters

ASSUMPTIONS

For developing the Economic Production model, we consider the following assumption:

The production process is random and follows a Weibull distribution, having the probability density function of the form

$$f(x)= \alpha\beta t^{\beta-1} e^{-\alpha(t)^{\beta}} ; \alpha > 0, \beta > 0, t > \gamma \quad \textbf{(1)}$$

Therefore, the instantaneous production rate at time 't' is

$$K(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1}, \text{ where } \alpha > 0, \beta > 0 \quad \textbf{(2)}$$

This production rate includes increasing /decreasing /constant rate of parameters, for different values of β.

The lifetime of commodity is also random, and follows a three parameter Weibull distribution, having probability density function of the form

$$g(t) = \theta\, \eta(t-\gamma)^{\eta-1} e^{-\theta(t-\gamma)^{\beta}} \quad \textbf{(3)}$$

Therefore, instantaneous rate of deterioration starts only after the time period of γ, this deterioration rate include increasing/decreasing/constant rates of deterioration, of various values of parameters.

- The demand rate λ(t) is linear function of unit demand rate and is of the form $\lambda(t)=\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$, where λ(t)>0, this demand rate includes the constant rate of demand.
- There is no lead time.
- Cycle length is fixed and known say, T.
- There is no repair or replacement of deteriorated items is thrown as an absolute.
- The money values remain constant through the period of production cycle
- The shortages allowed and fully backlogged.

With these assumptions, the stock level in the production system is initially zero, at time t=0, due to the production, the stock level increases with the production rate, during the period $(0, \gamma)$. Since, the deterioration starts after the time γ, during the period (γ, t_1) the stock level increases with a mix of production rate and deterioration rate at time t_1, the stock level reaches the maximum and production, is stopped during the period (t_1, t_2), the stock levels deteriorates with the deterioration rate and reaches to zero at time t_2, during the period (t_2, t_3), there is no production (or) deterioration. But, back orders are accumulating up to the time t_3, at time t_3, the production start again and fulfil the backlog demand with the production rate. The backlog demand is completely cleared, at time T and stock level reaches negative to zero.

INVENTORY MODEL WITH SHORTAGES

In this paper, we consider the production level inventory model, in which, shortages are allowed. The Production starts at time t=0, and the inventory level gradually increases with the passage of time due to production, after fulfilling demand and deterioration. The stock level reaches maximum at time t_1. The inventory decreases partly, due to demand and partly due to deterioration of items, during the time interval (γ,t_1). The shortages occurred during the time interval (t_2, t_3), are fully backlogged. The production recommences at time $t=t_3$, and backlogged demand is cleared during interval (t_3, T), and cycle continuous thereafter.

The schematic diagram, representing the instantaneous state of inventory is given in Figure 1

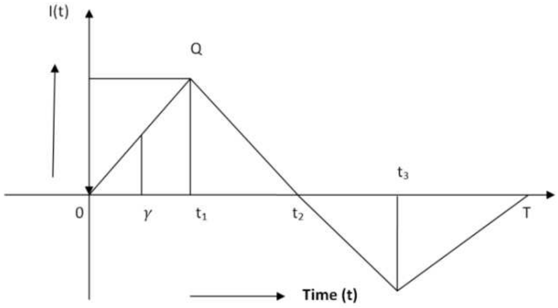


Figure 1: Instantaneous State of Inventory for Model with Shortages

Let I (t), denote the inventory level of the system at time 't' $0 \le t \le T$. The differential equations, describing the instantaneous states of I (t), in the interval (0, T) are

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; 0 < t \le \gamma \quad (4)$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; \gamma < t \le t_1 \quad (5)$$

$$\frac{d}{dt}I(t) + \theta\eta(t-\gamma)^{\eta-1}I(t) = \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; t_1 < t \le t_2 \quad (6)$$

$$\frac{d}{dt}I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \; ; t_2 < t \le t_3 \quad (7)$$

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; t_3 < t \le T \quad (8)$$

With the initial conditions, I(0)= 0, I(t_2)= 0, I(T)= 0

Solving the differential equations (4) to (8), using the boundary conditions, the instantaneous state of inventory at any time t, during the interval (0, T) is obtained as

$$I(t) = \alpha t^{\beta} - \frac{rt^{\frac{1}{n}}}{T^{\frac{1}{n}}} \; ; 0 < t \le \gamma \quad (9)$$

$$I(t) = \left[\left[e^{-\theta(t-\gamma)^{\eta}} \int_{\gamma}^{t} \left[\alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]\right] e^{\theta(t-\gamma)^{\eta}} dt + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right];$$

$\gamma < t \le t_1$ **(10)**

$$I(t) = \frac{r}{nT^{\frac{1}{n}}}\left[e^{-\theta(t-\gamma)^{\eta}}\int_{t_1}^{t} t^{\frac{1}{n}-1}e^{\theta(t-\gamma)^{\eta}}dt\right] + \left[\alpha t_1{}^{\beta} - \frac{rt_1{}^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]; t_1 < t \le t_2 \quad \textbf{(11)}$$

$$I(t) = \frac{r}{nT^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t^{\frac{1}{n}}\right]; t_2 < t \le t_3 \quad \textbf{(12)}$$

$$I(t) = r\left[1 - \left(\left(\frac{t}{T}\right)^{\frac{1}{n}}\right)\right] + \alpha(t^{\beta} - T^{\beta}); t_3 < t \le T \quad \textbf{(13)}$$

The stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t K(t)dt - \frac{\int_0^t rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}dt - I(t); 0 < t \le t_2$$

Implies

$$L(t) = \left[\alpha t^{\beta} - \frac{r\,t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right] - \left[\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1} - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]\right]dt + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]; \gamma < t \le t_1 \left[\alpha t_1^{\beta} - \frac{r\,t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right] - \left[\frac{r}{nT^{\frac{1}{n}}}\left[\int_{t_1}^{t} t^{\frac{1}{n}-1}dt\right] + \left[\alpha t_1{}^{\beta} - \frac{rt_1{}^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]; t_1 < t \le t_3$$

0, elsewhere

The stock loss due to deterioration in the interval (0, T) is

$$L(T) = \alpha t_1^{\beta} - r\left(\left(\frac{t_2}{T}\right)^{\frac{1}{n}}\right) \quad \textbf{(14)}$$

The total production in the cycle of length T is

$$Q = \alpha(t_1^{\beta} + T^{\beta} - t_3^{\beta}) \quad \textbf{(15)}$$

From equations (12) and (13), we get

$$I(t) = \frac{r}{nT^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t^{\frac{1}{n}}\right] \quad \textbf{(16)}$$

$$I(t) = r\left[1 - \left(\frac{t}{T}\right)^{\frac{1}{n}}\right] + \alpha(t^{\beta} - T^{\beta}) \quad \textbf{(17)}$$

When $t=t_3$, the equations (16) and (17) becomes

$$I(t_3) = \frac{r}{nT^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t_3{}^{1/n}\right] \quad \textbf{(18)}$$

$$I(t_3) = r\left[1 - \left(\left(\frac{t_3}{T}\right)^{\frac{1}{n}}\right)\right] + \alpha(t_3^{\beta} - T^{\beta}) \quad \textbf{(19)}$$

On equating these equations (18) and (19) and on simplification, we get t_2 in terms of t_3 as,

$$t_2 = t_3 + \left[(T^{n} - t_3) + \frac{nT}{r}\alpha(t_3^{\beta} - T^{\beta})^{n}\right] \quad \textbf{(20)}$$

Let K (t_1, t_2, t_3), be the total cost per unit. Since, the total cost is the sum of the set up cost per unit time, purchasing cost per unit time, holding cost per unit time and shortage cost per unit time. Then K (t_1, t_2, t_3) becomes

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{\gamma} I(t)dt + \int_{\gamma}^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right] + \frac{\pi}{T}\left[-\int_{t_2}^{t_3} I(t)dt - \int_{t_3}^{T} I(t)dt\right] \quad (21)$$

Substituting the values of I (t) and Q, given in equations, from (9) to (13) and (15) in equation (22), we have

$$K(t_1, t_3) = \frac{A}{T} + \frac{c}{T}\alpha\left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right)$$

$$+\frac{h}{T}\left[\left[\int_0^{\gamma} \alpha t^{\beta} - \frac{\gamma t^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt + \left[\int_{\gamma}^{t_1}\left[e^{-\theta(t-\gamma)^{\eta}}\left[\int_{\gamma}^{t}\left[\alpha\beta t^{\beta-1} - \frac{r t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right]e^{\theta(t-\gamma)^{\eta}}\right]dt + \left[\alpha\gamma^{\beta} - \frac{r\gamma^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]\right]\right]d + \left[\int_{t_1}^{t_2} e^{-\theta(t-\gamma)^{\eta}}\int_{t_1}^{t}\frac{r}{T^{\frac{1}{n}}}\left[(t - t_1)^{\frac{1}{n}}\right]e^{\theta(t-\gamma)^{\eta}}dt + \alpha t_1^{\beta} - \frac{r t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right]dt\right]$$

$$+\frac{\pi}{T}\left[-\left[\int_{t_2}^{t_3}\frac{r}{T^{\frac{1}{n}}}\left[t_2^{\frac{1}{n}} - t^{\frac{1}{n}}\right]dt\right] - \left[\int_{t_3}^{T} r\left[1 - \left(\left(\frac{t}{T}\right)^{\frac{1}{n}}\right)\right] + \alpha(t^{\beta} - T^{\beta})\right]dt\right] \quad (22)$$

On integrating and simplifying the equation (22), we get

$$K(t_1, t_3) = \frac{A}{T} + \frac{c}{T}\alpha\left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right)$$

$$+\frac{h}{T}\left[\left[\left[\frac{\alpha\gamma^{\beta+1}}{(\beta+1)}\right] - \left[\frac{n\gamma^{\frac{1}{n}+2}}{\left((n+1)T^{\frac{1}{n}}\right)}\right]\right] + \left[\left[\frac{\alpha(t_1-\gamma)^{\beta+1}}{(\beta+1)}\right] - \left[\frac{nr(t_1-\gamma)^{\frac{1}{n}+1}}{\left((n+1)T^{\frac{1}{n}}\right)}\right]\right] + \left[\frac{r}{T^{\frac{1}{n}}}\left[\frac{n}{n+1}(t_2 - t_1)^{\frac{1}{n}+1}\right] - 2t_1^{\frac{1}{n}}(t_2 - t_1) + \alpha t_1^{\beta}(t_2 - t_1)\right]\right]$$

$$+\frac{\pi}{T}\left[-\left[\frac{r}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}}(t_3 - t_2)\right) - \frac{n}{(n+1)}(t_3 - t_2)^{\frac{1}{n}+1}\right] - r\left[(T - t_3) - \frac{n}{(n+1)}(T - t_3)^{\frac{1}{n}+1}\right] + \alpha\left[\left[\frac{1}{(\beta+1)}(T - t_3)^{\beta+1}\right] - T^{\beta}(T - t_3)\right]\right] \quad (23)$$

OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this paper, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, t_3, we obtain the first order partial derivative of K (t_1, t_3), given in equation (23), with respect to t_1, t_3 and equate them to zero. The Condition for Minimization of K (t_1, t_2, t_3) is

$$|D| = \begin{vmatrix} \frac{\partial^2 K(t1,t3)}{\partial t_1^2} & \frac{\partial^2 K(t1,t3)}{\partial t_1 \partial t_3} \\ \frac{\partial^2 K(t1,t3)}{\partial t_1 \partial t_3} & \frac{\partial^2 K(t1,t3)}{\partial t_3^2} \end{vmatrix} > 0 \quad (24)$$

Where, D is the determinant of Hessian Matrix.

Differentiating K(t_1, t_3) with respect to t_1 and equating it to zero, we get

$$\frac{C}{T}\left[\alpha\beta t_1^{\beta-1}\right] + \frac{h}{T}\left[\left[\alpha t^{\beta} - \frac{r t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}}\right] + \left[-\frac{r t_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{2(n+1) r t^{\frac{1}{n}}}{nT^{\frac{1}{n}}} - \alpha t_1^{\beta}(\beta+1)\right]\right] = 0 \quad (25)$$

Differentiating $K(t_1, t_3)$, with respect to t_3 and equating it to zero, we get

$$\frac{C}{T}\left[-\alpha\beta t_3^{\beta-1}\right]+\frac{\pi}{T}\left[\left[-\frac{r}{T^{\frac{1}{n}}}\left[(t_2-t_3)^{\frac{1}{n}}\right]\right]+\left[r\left(1-\left(\frac{t_3}{T}\right)^{\frac{1}{n}}\right)+\alpha(T-t_3)^{\beta}\right]\right]=0 \qquad (26)$$

Solving the equations (25) and (26) simultaneously, we obtain the optimal time at which, the replenishment should be stopped i.e., t_1^* of t_1 and optimal time t_3^* of t_3, at which the replenishment is restarted after accumulation of backorders. The optimum ordering quantity Q^* of Q, in the cycle of length T is obtained, by substituting the optimal values of t_1 and t_3 in (2.3.15) as

$$Q^*=\alpha(t_1^{*\beta}+T^{\beta}-t_3^{*\beta}) \qquad (27)$$

NUMERICAL ILLUSTRATIONS

In this paper, we discuss the solution procedure of the model, through a numerical illustration by obtaining the production downtime, optimal selling price, optimal quantity and profit of an inventory system. Here, it is assumed that, the commodity is of the deteriorating nature and shortages are allowed, and fully backlogged. For demonstrating the solution procedure of the model, the deteriorating parameter 'α' is consider varying between 14 to 16, the values of the other parameters and costs associated with model are:

A= 190,200,210; C=14, 15, 16; T=12 months; α =14, 15, 16; β=0.49, 0.50, 0.51; γ=0.5, 1, 1.5; h = 0.1, 0.2, 0.3; n=1.5, 2, 2.5; $r = 13,14,17$; π= 0.45, 0.50, 0.55; θ=30, 40, 50; η= 0.20, 0.40, 0.60.

Substituting these values, the optimal selling price, optimal ordering quantity Q^*, replenishment time, optimal value of time and total profit are computed and presented in table 1

As the ordering cost 'A', increases from 190 to 210, the optimal values t_1^* at 1.063 is constant, the optimal replenishment t3* at 3.456 is constant, the optimal ordering quantity Q^* decreases from 39.543 to 39.540, the optimal profit rate increases from 74.796 to 76.455. When the cost per unit 'C', increases from 14 to 16, there is an increase in optimal ordering quantity Q^*, it increases from39.505 to 39.578, the optimal value of t_1^* increases from 1.062 to 1.064, the optimal replenishment time t_3^* decreases from 3.456 to 3.449, and the profit rate increases from 72.281 to 78.979.

If the deteriorating parameter 'α', increases from 14 to 16, there is an increase in optimal ordering quantity Q^*, from 36.882 to 42.183, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* decreases from 3.462 to 3.455, and the profit rate increases from 71.994 to 79.234. When 'β' is increasing from 0.49 to 0.51, there is increase in optimal ordering quantity Q^*, from 38.575 to 40.534, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* decreases from 3.463 to 3.450, and the profit rate increases from 74.255 to 77.037. The deterioration parameter 'γ', increases from 0.5 to 1.5, there is a decrease in optimal ordering quantity Q^* from 39.542 to 39.541, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* is constant at 3.456, and the profit rate increases from 75.599 to 75.662.

As the shortage cost per unit time 'π', increases from 0.45 to 0.55, there is an increase in optimal ordering quantity Q^*, from 39.531 to 39.552, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* decreases from 3.459 to 3.454, and the profit increases from 74.724 to 76.532.

As the indexing parameter ‘n’ is increasing from 1.5 to 2.5, there is a decrease in optimal ordering quantity Q^*, it decreases from 39.562 to 39.533, as the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t3*, increases from 3.451 to 3.459 and the profit rate is decreases from 81.054 to 73.531.

As the demand parameter ‘r’ is increased from 13 to 17, the optimal ordering quantity Q^* increases from 39.540 to 39.543, the optimal value of $t_1^*$is constant at 1.063, the optimal replenishment time t_3^* is decreases from 3.457 to 3.456, and profit rate increases 74.959 to 76.296. Inventory holding cost per unit time ‘h’, increases from 0.1 to 0.3, there is a decrease in the optimal ordering quantity Q^*, from 39.542 to 39.541, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* is constant at 3.456, and the profit rate increases from 75.298 to 75.957.

As the production parameter ‘θ’ increases from 30 to 50, there is an increase in optimal ordering quantity Q^* from 39.541 to 39.542, the optimal value of t_1^* decreases from 1.064 to1.062, the optimal replenishment time t_3^* increases from 3.455 to 3.457, and profit rate increases from 75.640 to 75.612. When ‘η’ is increasing from 0.20 to 0.60, there is an increase in optimal ordering quantity Q^*, the increase from 38.519 to 40.632, the optimal value of t_1^* is constant at 1.063, the optimal replenishment time t_3^* is constant at 3.456, and the profit rate increases from 74.111 to 76.323.

Table 1: Optimal Values of t1*, t3*, Q*, K* for Different Values of Parameters with Shortages

A	C	T	α	β	π	H	n	r	Υ	θ	η	t_1^*	t3*	Q*	K*
200	15	12	15	.5	.5	0.2	2	15	1	50	0.40				
190												1.062	3.458	39.540	76.455
200												1.063	3.456	39.542	75.627
210												1.064	3.455	39.543	74.796
	14											1.064	3.452	39.578	72.282
	15											1.063	3.456	39.542	75.627
	16											1.062	3.449	39.505	78.979
			14									1.064	3.455	36.882	71.994
			15									1.063	3.456	39.542	75.627
			16									1.062	3.462	42.183	79.234
				0.49								1.064	3.450	38.575	74.255
				0.50								1.063	3.456	39.542	75.627
				0.51								1.062	3.463	40.534	77.037
					0.45							1.062	3.454	39.531	74.724
					0.50							1.063	3.456	39.542	75.627
					0.55							1.064	3.459	39.552	76.532
						0.1						1.061	3.459	39.542	75.298
						0.2						1.063	3.456	39.542	75.627
						0.3						1.065	3.454	39.541	75.957
							1.5					1.060	3.459	39.533	73.531
							2					1.063	3.456	39.542	75.627
							2.5					1.067	3.452	39.562	81.054
								13				1.061	3.455	39.543	75.559
								15				1.063	3.456	39.542	75.627
								17				1.064	3.459	39.540	76.296
									0.5			1.061	3.455	39.541	75.599
									1			1.063	3.456	39.542	75.627
									1.5			1.065	3.458	39.544	75.662
										30		1.064	3.455	39.540	75.640
										40		1.063	3.456	39.541	75.627
										50		1.062	3.457	39.542	75.612
											0.20	1.065	3.452	38.519	74.111
											0.40	1.063	3.456	39.542	75.627
											0.60	1.061	3.459	40.632	76.381

SENSTIVITY ANALYSIS OF THE MODEL

Sensitivity analysis, is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%,-10%,-5%,0%,5%,10%,15%), at a time for the model under study. The results are presented in table 2.

As the ordering cost ‘A’ decreases, the optimal value of t_1^* is constant, the optimal replenishment time t_3^* decreases, the optimal ordering quantity Q^* increases and profit rate K^* are decreases. If ‘A’ increases, the optimal value of

t_1^* is constant, the optimal replenishment time t_3* increases, the optimal ordering quantity Q^* decreases and profit rate K^* increases

As the cost per unit 'C' is decreased, the optimal value of t_1^* decreases, the optimal replenishment time t_3^* decreases, the optimal ordering quantity Q^* decreases and the profit rate K^* decreases. If 'C' increase, the optimal value of t_1^* increases, the optimal replenishment time t_3^* increases, the optimal ordering quantity Q^* increases and the profit rate K^* increases.

As the deteriorating parameter 'α 'decreases, the optimal value of t_1^* decreases, the optimal replenishment time t_3^* increases, the optimal ordering quantity Q^* and the profit rate K^* increases. If 'α' decreases, the optimal value of t_1^* increases, the optimal replenishment time t_3^*, the optimal ordering quantity $Q^*$and profit rate K^*, decreases.

As 'β' decreases, the optimal value of t_1^* increases, the optimal replenishment time t_3^* increases, the optimal ordering quantity Q^* and profit rate K^*, decreases. If 'β' increases, the optimal value of t_1^* decreases, the optimal replenishment time t_3 decreases, the optimal ordering quantity Q^* and profit rate K^* increases.

As the indexing parameter 'n' decreases, the optimal value of t_1^* remains constant, the optimal replenishment time t_3^*decreases and profit rate K^* decreases, the optimal ordering quantity Q^* increases. If 'n' increases, the optimal value of t_1^* remains constant, the optimal replenishment time t_3^* increases and profit rate K^* increases, the optimal ordering quantity Q^* decreases

As the demand parameter 'r' decreases, the optimal value of t_1^* remains constant, the optimal replenishment time t_3^* increases and the optimal ordering quantity Q^* decreases and the profit K^* rate decreases. If 'r' increases, the optimal value of t_1^* remains constant, the optimal replenishment time t_3^* decreases and the optimal ordering quantity Q^* increases and the profit rate K^* increases.

When the shortage cost per unit 'π' decreases, the optimal value of t_1^* remains constant, the optimal replenishment t_3^* increases, the optimal ordering quantity Q^* and the profit rate K^* decreases. If 'π' increases, the optimal value of t_1^* remains constant, the optimal replenishment t_3^* decreases, the optimal ordering quantity Q^* and the profit rate K^* increases.

As the holding cost per unit 'h' decreases, the optimal value of t_1^* remains constant, the optimal replenishment t_3^* remains constant, the profit rate K^* decreases, and the optimal ordering quantity Q^* increases. If the holding cost per unit 'h' increases, the optimal value of t_1^* remains constant, the optimal replenishment t_3^* remains constant, the profit rate K^* increases, and the optimal ordering quantity Q^* decreases.

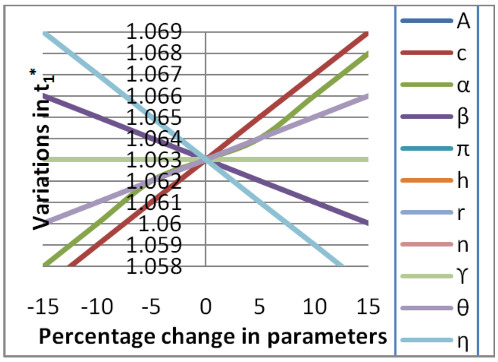
As the deterioration parameter 'γ' decreases, the optimal value t_1^* is constant, the optimal replenishment t_3^* is constant, the profit rate K^* decreases, and the optimal ordering quantity Q^* increases. If 'γ' increases, the optimal value of t_1^* is constant, the optimal replenishment t_3^* is constant, the profit rate K^* increases, the optimal ordering quantity Q^* decreases.

As the production parameter 'θ' decreases, the optimal value of t_1^* decreases, the optimal replenishment t_3^* decreases, the optimal ordering quantity Q^* increases and the profit rate K^* decreases. If 'θ' increases the optimal value of t_1^* increases, the optimal replenishment t_3^* increases, the optimal ordering quantity Q^* decreases and the profit rate K^* increases.

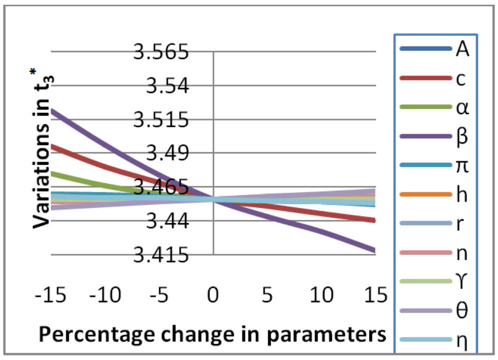
As 'η' decreases, the optimal value of t_1^* increases, the optimal replenishment t_3^* increases, the optimal ordering quantity Q* increases and the profit rate K* increases. If 'η' increases, the optimal value of t_1^* decreases, the optimal replenishment t_3^* decreases, the optimal ordering quantity Q^* decreases and the profit rate K^* decreases.

Table2: Sensitivity Analysis of the Model –with shortages

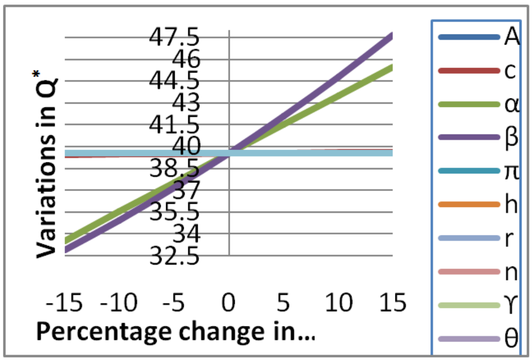
Variation Parameters	Optimal Policies	Change in Parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
A=200	t_1^*	1.0633	1.0633	1.0633	1.0633	1.0633	1.0633	1.0633
	t3*	3.455	3.456	3.456	3.456	3.457	3.457	3.457
	Q*	39.546	39.545	39.543	39.542	39.540	39.538	39.537
	K*	73.155	73.966	74.796	75.627	76.458	77.289	78.120
C=15	t_1^*	1.061	1.062	1.063	1.063	1.064	1.065	1.065
	t3*	3.473	3.468	3.462	3.456	3.451	3.445	3.440
	Q*	39.458	39.486	39.514	39.542	39.569	39.595	39.622
	K*	68.105	70.609	73.117	75.627	78.141	80.657	83.176
α =15	t_1^*	1.062	1.062	1.063	1.063	1.063	1.063	1.064
	t3*	3.475	3.466	3.460	3.456	3.455	3.454	3.453
	Q*	33.537	35.546	37.548	39.542	41.525	43.496	45.455
	K*	67.423	70.169	72.905	75.627	78.335	81.005	83.696
β=0.5	t_1^*	1.064	1.063	1.063	1.063	1.062	1.061	1.059
	t3*	3.521	3.496	3.474	3.456	3.443	3.434	3.432
	Q*	32.906	34.964	37.174	39.542	42.071	44.765	47.626
	K*	66.214	69.130	72.266	75.627	79.221	83.051	87.119
π =0.5	t_1^*	1.063	1.063	1.063	1.063	1.063	1.063	1.063
	t3*	3.460	3.459	3.458	3.456	3.455	3.454	3.452
	Q*	39.525	39.531	39.536	39.542	39.547	39.552	39.558
	K*	74.272	74.724	75.175	75.627	76.080	76.532	76.984
h =0.2	t_1^*	1.063	1.063	1.063	1.063	1.063	1.063	1.063
	t3*	3.456	3.456	3.456	3.456	3.456	3.456	3.456
	Q*	39.542	39.542	39.542	39.542	39.541	39.541	39.541
	K*	75.529	75.562	75.595	75.627	75.660	75.693	75.726
r =15	t_1^*	1.063	1.063	1.063	1.063	1.063	1.063	1.063
	t3*	3.457	3.457	3.457	3.456	3.456	3.456	3.456
	Q*	39.540	39.541	39.541	39.542	39.542	39.542	39.543
	K*	74.875	75.126	75.377	75.627	75.878	76.129	76.380
n=2	t_1^*	1.063	1.063	1.063	1.063	1.063	1.063	1.063
	t3*	3.454	3.455	3.456	3.456	3.457	3.457	3.458
	Q*	39.552	39.548	39.544	39.542	39.539	39.537	39.535
	K*	78.136	77.123	76.302	75.627	75.064	74.589	74.184
γ =1	t_1^*	1.063	1.063	1.063	1.063	1.063	1.063	1.063
	t3*	3.456	3.456	3.456	3.456	3.456	3.456	3.456
	Q*	39.542	39.542	39.542	39.542	39.542	39.542	39.541
	K*	75.618	75.621	75.624	75.627	75.631	75.634	75.637
θ =50	t_1^*	1.059	1.061	1.062	1.063	1.064	1.065	1.066
	t3*	3.450	3.452	3.454	3.456	3.455	3.460	3.462
	Q*	39.552	39.548	39.546	39.542	39.538	39.535	39.532
	K*	75.608	75.611	75.614	75.627	75.630	75.633	75.635
η =0.60	t_1^*	1.067	1.066	1.065	1.063	1.063	1.061	1.060
	t3*	3.458	3.457	3.457	3.456	3.455	3.454	3.453
	Q*	39.550	39.549	39.547	39.542	39.537	39.534	39.531
	K*	74.865	75.106	75.367	75.627	75.868	76.109	76.370



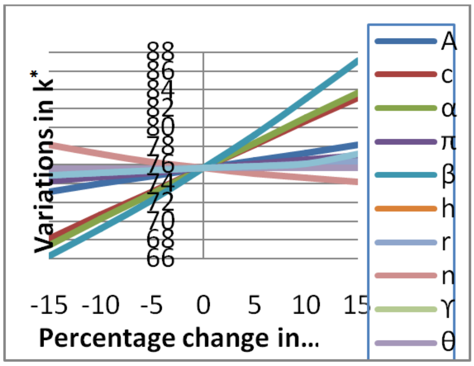
(a)



(b)



(c)



(d)

Figure 2: Relationship between Parameters and Optimal Values with Shortages

CONCLUSIONS

An EPQ model for deteriorating items, was developed and analyzed with the assumption that, production rate is a function On-hand inventory, demand is a power function of time, and lifetime of the item is random and follows a three parameters, Weibull distribution. Here, it is assumed that, the demand rate is of the form $\lambda(t)=\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$, where 'r' is the size of demand over the period (0,T) and 'n' is the pattern index. For different values of the indexing parameter, the demand pattern includes co- increasing and decreasing rates, of demand. Assuming that, shortages are allowed and fully backlogged, and using the differential equations, the instantaneous state of inventory, the backlogged demand and the maximum shortages levels, were obtained. With suitable cost considerations, the total cost function is derived. By minimizing the total cost function, the optimal ordering quantity, optimal production downtime and optimal production uptime, were obtained.

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